THEORY CLUB : DONUT PROBLEM SESSION

(1) After a difficult problem session, Shyamal wants to eat a donut, which is indefinitely rolling in a straight line. His speed is at most the speed of the donut. Assuming that the world is flat (not round), where can he stand such that the donut can eventually be eaten?

(2) (a) Daniel wants to test the durability of his donuts. There are 2 donuts that will burst when dropped at any floor above some integer x ($0 \leq x \leq 100$) of a 100-story building. What is the minimum number of drops needed to guarantee the value of x using at most 2 donuts? (Note: It is possible for a donut to break when dropped from floor 1 or not break when dropped from floor 100).

(b) Given an n-story building and k donuts, is there an algorithm to find the minimum number of drops needed to guarantee the value of x? ($0 \leq x \leq n$) (brilliant.org)

(3) (a) Sherry wants to hand out doughnuts to Theory Club according to some probability distribution (random variable X). But she?s very nitpicky about how to do so. She wants the expected number of doughnuts handed out to be 2 ($\mathbb{E}(X) = 2$) and the expected (number of doughnuts) squared to be 3 (meaning $\mathbb{E}(X^2) = 3$). Because she's a mage, she has magical powers like the ability to cut doughnuts into arbitrarily sized pieces and make negative doughnuts. Can she with all her magical powers do this? (*Hint:* Prove $Var(x) = \mathbb{E}(x^2) - (\mathbb{E}(x))^2$).

(b) What if she wants to distribute doughnuts, random variable X, from some distribution D such that

$$\mathbb{E}[X] = 1, \mathbb{E}[X^2] = 2, \mathbb{E}[X^3] = 4, \mathbb{E}[X^4] = 5.$$

(c) Given $2n$ moments come up with a necessary condition for Sherry to be able to distribute doughnuts. Can you design an efficient (polytime) algorithm to check this condition given $2n$ moments?

(4) Given a sequence of donuts labeled $1, 2, \ldots, n$ one may interchange two consecutive blocks to obtain a new ordering. For instance,

$$3, 5, 4, 8, 9, 7, 2, 1, 6$$

can be transformed to

$$3, 9, 7, 2, 5, 4, 8, 1, 6$$

Date: January 14, 2019.

by swapping the consecutive blocks $5, 4, 8$ and $9, 7, 2$. Find the least number of changes required to change

$$n, n-1, n-2, \ldots, 1$$

to

$$1, 2, \ldots, n.$$

(Source: Bulgaria 2001)

(5) There are 2000 donuts, every two of which were initially joined by a wire. The hooligans Arvind and Chunlok cut the wires one after another. Arvind, who starts, cuts one wire on his turn, while Chunlok cuts two or three. A donut is said to be disconnected if all wires incident to it have been cut. The player who makes some donut disconnected loses. Who has a winning strategy? (Source: Russia 1999)

(6) 100 rooms each contain countably infinite boxes labelled with natural numbers. Each box contains a real number of donuts, but this number is same in each of the 100 rooms (i.e if Room 1, Box 5 has 3.2 donuts and Room 2, Box 5 has 3.2 donuts, etc.) 100 Theory club members will play a game. Each will go in a room and open as many boxes as they like. Then, each Theory club member will guess the number in an unopened box in their room. They win if 99 of them guess correctly. What is the winning strategy? [Credit to Patrick Yee from "Actually Good Math Problems" on FB]