Graphs

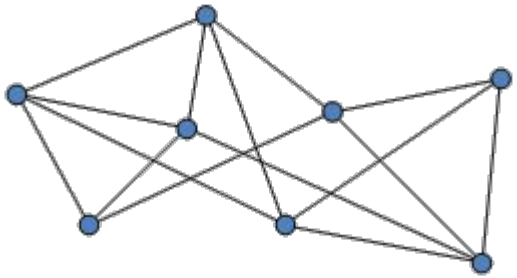
Arvind Ramaswami

Outline

- What is a graph?
- Properties of graphs
- Graph algorithms
- Well-known graph problems
- Problem set

What is a graph?

- A collection of vertices and edges G(V, E)

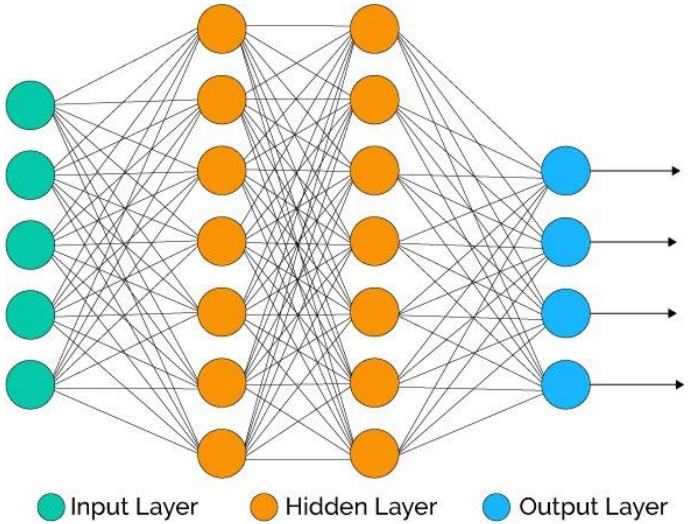


Why graphs?

- Show relationships between different entities (e.g. people)

Other applications

- Neural networks (machine learning)



Definitions

- The **degree** of a vertex $v \in V$ is the number of vertices that are incident to v.
- Cycle
- Tree

Graph Algorithms - Short Cycle Decompositions

- Can we find an upper bound the length of the smallest cycle in a graph?
- Create an algorithm to decompose a general graph into O(n) edges and cycles of length at O(log n), and analyze the running time.

Give an O(m log^3 n) time or faster algorithm for finding short cycle decompositions of length O(log n).

Well-known problems

Graph isomorphism problem

State-of-the-art algorithms have complexity exp(n^{1/2} + O(1)) -- Babai 1983

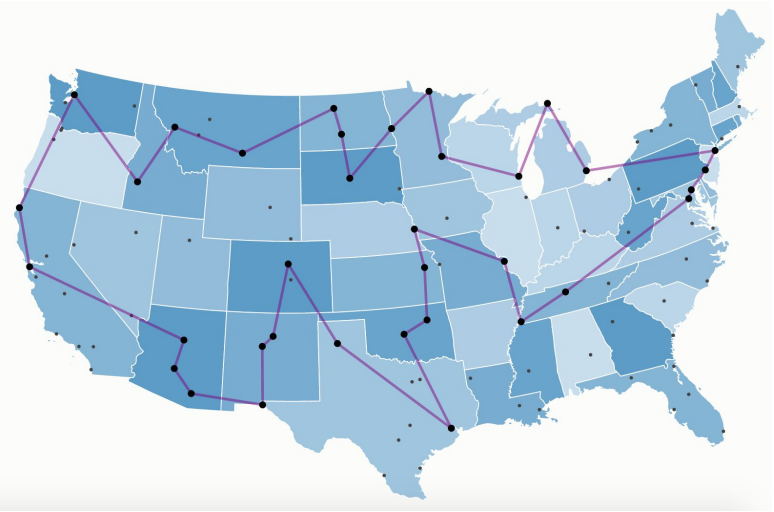
Open problems:

- Can the exponent of n be reduced? (less than 1/2)
- can this be done in polynomial time?

Well-known problems

Traveling salesman problem

How do you compute the shortest path to visit all states?



Topic in graph theory: Shortest-path and applications in graph algorithms

- Can we find an upper bound the length of the smallest cycle in a graph?
- Create an algorithm to decompose a graph into edges and cycles of length at O(log n), and analyze the running time.

Exercise: Any undirected graph with minimum degree at least 3 has a cycle of length at most log n, and such a cycle can be found in O(m) time.

References

Richard Peng CS7510 Webpage (Graph algorithms)

Olympiad Combinatorics by Pranav Sriram: Chapter 7

Conferences:

STOC, SODA, WADS