1. For a graph $G = (V, E)$, denote $d(v)$ to be the *degree* of a vertex v ($v \in V$), where the degree of a vertex is the number of edges incident to it. Show that for any graph $G = (V, E)$,

$$\sum_{v \in V} d(v) = 2|E|$$

2. (a) A perfect matching in a graph $G = (V, E)$ is a set of disjoint edges $M \subseteq$ such that there is an edge incident to every vertex of G. A bipartite graph is a graph in which there is a partition of a vertex set $V = A \cup B$ such that every edge is incident to one vertex in A and one vertex in B. What conditions do we have to have on A and B if there is a perfect matching?

   (b) Use the previous part to show that you cannot tile an 8x8 chessboard with opposite corners removed using 2x1 tiles.

3. Suppose that for $G = (V, E)$, $max_{v \in V} deg(v) \leq \Delta$. Show that V can be colored with at most $\Delta + 1$ colors, such that no two neighboring vertices are of the same color.

4. In a graph G with n vertices and m edges, show that there exists an induced subgraph H with each vertex having degree at least $\frac{m}{n}$.

5. Let S be a set of n points in the plane such that the greatest distance between two points of S is 1. Show that at most n pairs of points of S are at distance 1 apart.

6. Show that it is possible to partition the vertex set V of a graph G on n vertices into two sets V_1 and V_2 such that any vertex in V_1 has at least as many neighbors in V_2 as in V_1, and any vertex in V_2 has at least as many neighbors in V_1 as in V_2.

7. A town has 3n citizens. Any two persons in the town have at least one common friend in this same town. Show that one can choose a group consisting of n citizens such that every person of the remaining 2n citizens has at least one friend in this group of n.

8. Let T be a tree with t vertices, and let G be a graph with n vertices. Show that if G has at least $(t-1)n$ edges, then G has a subgraph isomorphic to T.