# Linear Programs

#### What is a Linear Program?

 $\begin{array}{ll} \mbox{maximize} & x_1+x_2\\ \mbox{subject to} & & \\ & x_1+2x_2 \leq 1\\ & 2x_1+x_2 \leq 1\\ & x_1 \geq 0\\ & x_2 \geq 0 \end{array}$ 



 $\begin{array}{ll} \text{maximize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & \\ & A \mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$ 



#### An Example Problem

You are running a paint shop. You produce , blue, orange, and purple. You mix the following to make the following colors:

- Orange: 1 part primer, 2 parts red color and 4 parts plaster
- Green: 2 parts primer, 1 part blue color, and 2 parts plaster
- Purple: 1 parts blue, 1 part red, 2 parts primer, 4 parts plaster

Given a budget, the price of each of the raw materials to buy (per unit volume), and the cost at which you sell each color (per unit volume), what should you buy and what paints should you make to maximize your profit?

# Duality!

How do we know when we are done?

$$\begin{array}{cccc} \max & \mathbf{c}^T \mathbf{x} & \min & \mathbf{c}^T \mathbf{y} \\ \text{subject to} & & & \text{subject to} \\ & & & A\mathbf{x} \leq \mathbf{b} \\ & & & \mathbf{x} \geq \mathbf{0} & & & \mathbf{y} \geq \mathbf{0} \end{array}$$

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### 1. A Geometric View: Polytopes

A polytope is a bounded region that can be described as the intersection of finitely many halfspaces.

### 2. Vertex Solutions

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It's nice to not have wiggle room sometimes.

Linear Programs and Approximation Algorithms

Vertex Cover Load Balancing



## Total Unimodularity

When do Linear Programs always have integer solutions?



# Thanks!

Any questions?

#### **Credits**

- Presentation template by <u>SlidesCarnival</u>
- Photographs by <u>Unsplash</u>
- http://theory.stanford.edu/~trevisan/cs2 61/lecture05.pdf
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