1. **Problem 1.** Suppose you are given a linear program

$$\max c^T x$$

$$Ax \leq b$$

$$x \geq 0$$

   And you know that x and y are primal and dual optimal. Then show that for any i either $(Ax)_i = b_i$ or $y_i = 0$.

2. **Problem 2.** Let $\mathcal{P}$ be a polytope. Then recall that a point v is a vertex if we have that whenever $v = \frac{1}{2}(x+y)$ with $x, y \in \mathcal{P}$ then $x = y = v$. Prove that every polytope has a vertex.

   (Hint: Consider the point $x \in \mathcal{P}$ that maximizes $||x||$.)

3. **Problem 3.** For any polytope $\mathcal{P}$, prove the value of $c^T x$ is maximized on a vertex. (Note: there can be many points where the maximum point is achieved but at least 1 is a vertex solution.)

4. **Problem 4.** Consider a polytope given by $\mathcal{P} = \{x : Ax \leq b, x \geq 0\} \subset \mathbb{R}^d$. Prove that if x is a vertex of the polytope then we must have that at least d of the equations $Ax \leq b$ and $x \geq 0$ must hold with equality.

5. **Problem 5.** Prove that if the value of primal LP is unbounded then dual LP is unfeasible.

6. **Problem 6.** Formulate the following as an LP

$$\max |c^T x|$$

$$Ax \leq b$$

$$x \geq 0$$

7. **Problem 7.** Formulate the following as an linear program.

$$\min ||Ax - b||_1$$

$$||x||_\infty \leq 1$$

   where $||x||_1 = \sum_i |x_i|$ and $||x||_\infty = \max_i |x_i|$.

   (Source: http://www.seas.ucla.edu/ vandenbe/ee236a/homework/problems.pdf)

8. **Problem 8.** Consider the fractional vertex cover problem for a graph $G = (V, E)$:

$$\min_{v \in V} x_v$$

$$\forall (u, v) \in E : x_u + x_v \geq 1$$

$$x_v \geq 0$$

Prove that there exists a half-integral optimal solution $x \in \{0, 1/2, 1\}^{|V|}$.

9. **Problem 9.** Extend the approximation algorithm given for vertex cover to one for a weighted vertex cover. That is for positive costs c_v find an approximate solution to:

$$\sum_{v \in V} c_v x_v$$

$$\forall (u, v) \in E : x_u + x_v \geq 1$$

$$x_v \in \{0, 1\}$$