Graphs and Algorithms

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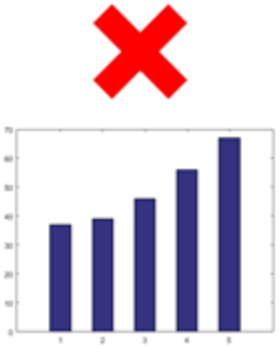
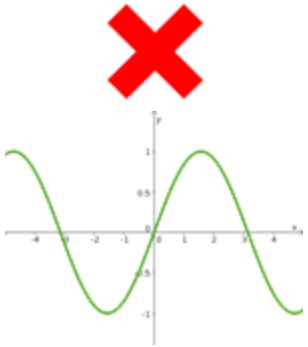
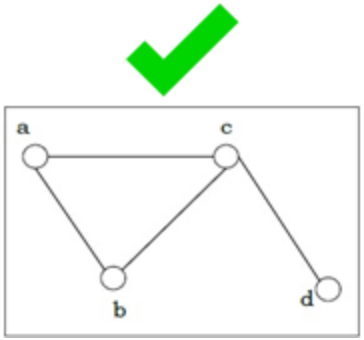
Big O Theory Club

What is a Graph?

- In discrete math, a graph is a collection of vertices and edges.
- An edge connects two vertices.







Examples of Graphs

Road map across the US

Social Network

How do we describe a graph?

Computer sees:

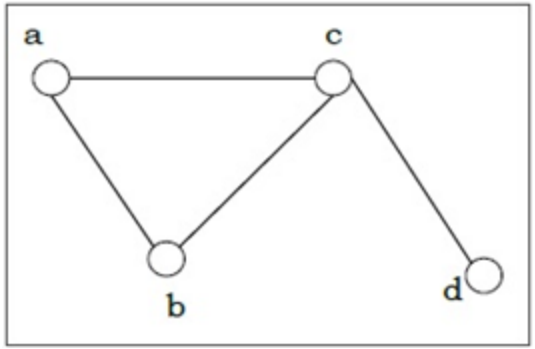
Edge List

- a: b, c
- b: a, c
- c: a, b, d
- d: c

Adjacency Matrix

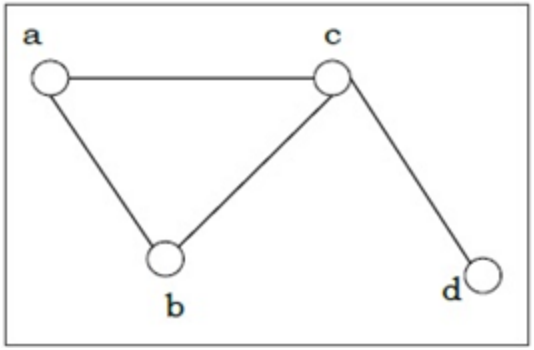
	a	**b**	**c**	**d**
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

We see:



Properties of Graphs

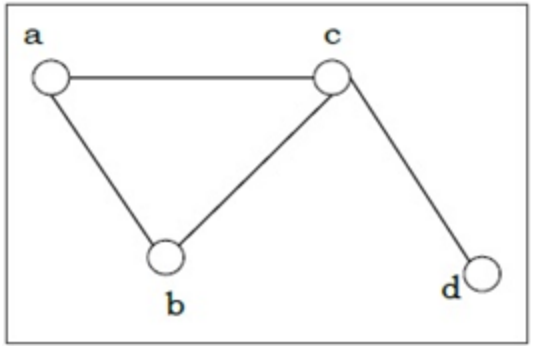
- The **degree** of a vertex is the number of edges containing that vertex.
  - Degree of **a** is 2, since (a,b) and (a,c) are edges.
- A **path** is a sequence of distinct vertices where there is an edge between every two consecutive vertices in the sequence.
- A **cycle** is a path that ends where it starts.
  - Cycle of 3: (a, b, c)
- The **distance** between two vertices is the length of the shortest path between them.
  - Length of a path = number of edges it contains



"First Theorem of Graph Theory"

Theorem: In a graph *G*, the sum of the degrees of the vertices is equal to twice the number of edges.

Proof:



"First Theorem of Graph Theory" Proof

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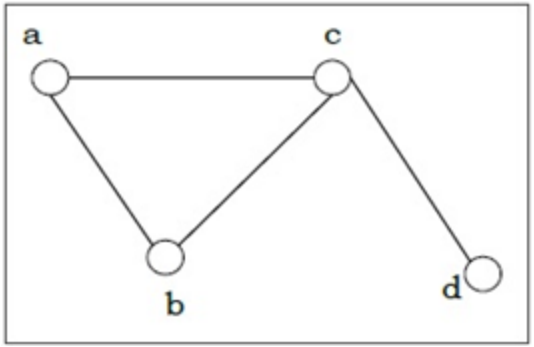


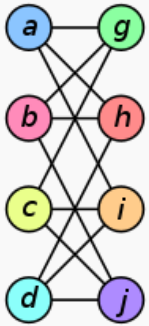
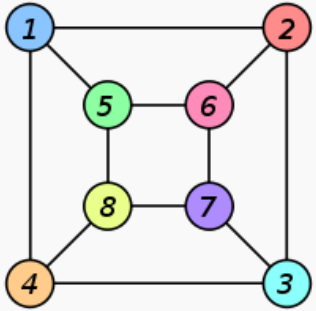
Proof: Let our graph *G* have *m* edges and *n* vertices $v_1, v_2, \ldots, v_n$ We have that:

$$\sum_{i=1}^{n} \deg(v_i) = 2m$$

On the left we sum each edge twice, because an edge (v_i, v_j) is counted in the degree of v_i and v_j.

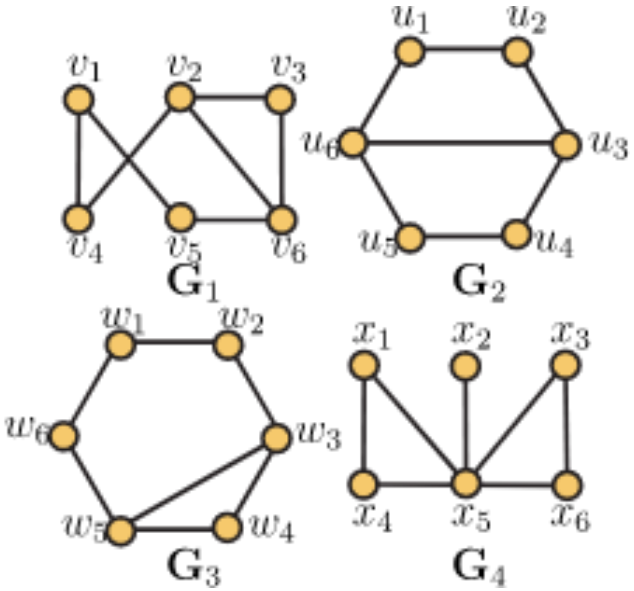
Graph Isomorphism Example

- Two graphs *G* and *H* are **isomorphic** if there exists a mapping *f* between the vertices of *G* and *H* such that the edges are preserved.

Graph G	Graph H	An isomorphism between G and H
		$f(a) = 1$ $f(b) = 6$ $f(c) = 8$ $f(d) = 3$ $f(g) = 5$ $f(h) = 2$ $f(i) = 4$ $f(j) = 7$

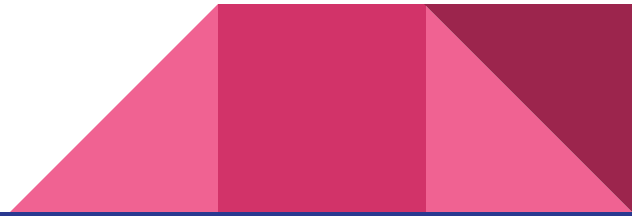
Graph Isomorphism Example

- Which of these four graphs are isomorphic? Why or why not?
- Hint: Graph invariants!



Graph Isomorphism Problem

- Given two finite graphs ***G*** and ***H*** with ***n*** vertices, determine if ***G*** and ***H*** are isomorphic.
- Brute force algorithm: runtime of $O(n!n^2)$
- **Open problem:** Can this problem be solved in polynomial time?
  - Currently not known to be P or NP-Complete



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