Theory Club - First Meeting

Discord

Attendance

Introductions

- Brian Zhang
- Christian Engman
- Edward Chen
- Akhil Kammila

General Information

General Meetings

- Weekly talks/discussions
- Doughnut problem sessions
- Themed activity nights

Goals

- See CS theory outside of school
- Get students interested or involved in CS research

Prerequisites

- No theory background needed
- No coding

The Poker Chip Game

Rules of Fibonacci Nim

- Whoever takes the last chip wins
- Can take no more than double what the opponent took on their last move
- Cannot take all chips on the first move

Breaking an Integer into Fibonacci Numbers

- We can break an integer into Fibonacci numbers by continually taking the largest possible

- **Observation**: We never select consecutive Fibonacci numbers

The Winning Strategy

- N = number of remaining chips
- Decompose N into Fibonacci numbers and take the smallest, if possible

- Quiz: If there are 20 chips, and I am making the first move, how many should I take?

Some Informal Terminology

- Let **smallest Fibonacci of N** refer to the smallest element of the greedy decomposition of N
- Let a **winning state** refer to when a player is about to move and the limit is above the smallest Fibonacci of the pile's remaining size (i.e. limit is high enough so we can execute our strategy)
- Let a **losing state** refer to a position that is not a winning state (i.e. we can't execute our strategy)

Proof Sketch - Claim 1

- Claim: If a player is in a losing state, they cannot take all the remaining chips

Claim 1

- To immediately win, a player needs to take the whole remaining pile (say of size N)

- Limit < Smallest Fibonacci of N <= N
- Limit is too low in an uncomfortable position to win immediately

Proof Sketch - Claim 2

- Claim: From a winning state, a player can return a losing state to their opponent by taking the smallest Fibonacci

Note the following:

- If N = F1 + F2 + F3…, N - F1 decomposes to F2 + F3…
- Chip limit is now 2 * F1 after taking F1
- Fibonacci numbers in our decomposition are never consecutive in the Fibonacci sequence

Claim 2

- Suppose N = F_0 + F_1 + . . .
- N - F_0 = F_1 + F_2 + . . . is the decomposition of the new pile size
- Smallest Fibonacci now is F_1
- Limit is now 2 * F_0
- Since we require our Fibonacci numbers to be nonconsecutive, F_1 > 2 * F_0.
  - Why is this true?

Claim 3

- Claim: From a losing state, we cannot take all the chips. And any amount of chips we take will give the opponent a winning state

Observations to Maybe Consider:

- Suppose we could give the opponent a losing state by taking x from N
- So the smallest Fibonacci in N-x would be > 2x
- We could then safely build the decomp of N by taking the decomp of N-x and appending the decomp of x
- Does this contradict our original state being a losing state?

Claim 3

- Proceed with a proof by contradiction
- Suppose for an x < limit, the smallest Fibonacci of N - x > 2 * x
- Take the Zeckendorf decompositions of N - x and x to make a representation of N
  - Why is this a valid Zeckendorf decomposition of N?

- Smallest Fibonacci in this combined representation is at most x, since it includes the decomposition of x
- But x < limit, so our original position was not a losing state (contradiction)

Putting it Together

- If the initial state is a winning state, and you go first…
- By continually taking the smallest Fibonacci, you can force your opponent to always be in a losing state
- From such a position, your opponent cannot take the last chip. So you will always win

Proof Sketch

- Claim 1: If a player is in a bad state, they cannot take all the remaining chips

- Claim 2: From a comfortable position, a player can return an uncomfortable position to their opponent by taking the smallest Fibonacci

- Claim 3: From an uncomfortable position, any move returns a comfortable position for the opponent
    - Hint: Try a proof by contradiction

The Circular Table Game

Rules

- Two players take turns placing pennies on a circular table
- Each player must place a penny so that there is no overlap and no hanging off the table
- Player loses if they are unable to place a penny

Winning Strategy

- Which player has a winning strategy?
- What is their winning strategy?

Brief Overview of CS Theory

What Does CS Theory Consider?

Algorithms

- Finding an optimal route to visit a set of cities
- Selecting the best candidate from a stream of applicants
- How to encrypt and decrypt data with a public/private key pair

Limits of Computation

- How many comparisons are needed to sort a list
- Can randomness make us compute things faster
- If a problem is hard, what other problems must be hard

Tools Commonly Used

- Discrete Math
  - Number theory
  - Graph theory
- Continuous Math
  - Calculus
  - Geometry
- Algorithmic Ideas
  - Data structures
  - Dynamic programming

Open Problems

P vs. NP

If we can check a solution quickly, can we build a solution quickly?

Fast Integer Factorization

Can we prime factorize an integer efficiently on a classical computer?

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